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An Envelope Process for Multifractal Traffic Modeling

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Abstract

In this paper, a novel envelope process for multifractal traffic modeling is introduced. The envelope process is an upper bound for the amount of work arrived in a multifractal Brownian motion process. The time scale of interest of a queueing system fed by a multifractal stream is computed. Simulation experiments using both real and synthetic data show that the proposed model is accurate.

1 Introduction

Since the seminal work of Leland et al [5], several studies have shown that network traffic presents scale invariance, or "scaling", which is the absence of any specific time scale at which the "burstiness" of a traffic stream can be characterized. Instead, it is necessary to describe the traffic across different time scales. Self-similar or (mono) fractal processes have been used for modeling network traffic since then.

Scaling of fractal traffic is defined by a single constant value: the Hurst parameter, H. One of the most popular fractal processes for traffic modeling is the Fractal Brownian Motion process (fBm) due to its parsimonious representation of the modeled traffic. fBm is an accurate model when: i) the traffic results from the aggregation of several sources streams with low activity compared to the link bandwidth, ii) the impact of flow control is not relevant and iii) the time scale of interest is within the scaling region. The multifractal Brownian motion (mBm) is the multifractal generalization of the fractal Brownian motion. mBm is a Gaussian process which is able to capture the high variability existing at small time scales. It has the nice property that at small time scales (locally) its realization can be described by an fBm.

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Both Internet Protocol (IP) and Variable Bit Rate (VBR) video traffic present non-trivial scaling structure at small scales in addition to long memory [2]. At small scale, traffic is highly variable, more complex and follows less definitive scaling laws. For these traffics the marginal distribution of counts is clearly non-Gaussian, calling for a representation beyond second-order statistics. Moreover, the scaling exponent of the variance on time scale shorter than a typical (cut-off) one is smaller than an asymptotic exponent.

If on one hand, at the network core long term correlations due to traffic aggregation (additive property) are more important than the variability at small scales [1]. On the other hand, at the network edge the variability at small time scale (multiplicative property) [4] plays a major role. These patterns can be modeled by multifractal processes which capture both long memory and high variability at small scales.

Some studies have claimed that the multiscaling nature of IP traffic is highly influenced by the Transmission Control Protocol (TCP) congestion control mechanism rather than solely by network-related variability, such as the diversity of link capacity in the Internet [4]. Nonetheless, the multiplicative origin of IP traffic remains to be fully explained. Understanding the interaction between TCP congestion control and IP multiscaling is undoubtfully relevant for predictive purpose. However, this work is concerned with open loop aspects of IP traffic for network dimensioning.

Solving queueing systems with (multi/mono) fractal input is not a trivial task. Large Deviation theory can be employed to overcome such difficulty. However, it implies in making non-realistic assumption about the buffer size. Envelope processes are upper bounds to the accumulated amount of work (traffic) arrived from a process up to a certain time. Envelope processes are parsimonious representations of stochastic processes and allow simple solutions for queueing systems fed by (mono/multi) fractal processes which do not incorporate any assumption about the buffer size.

The major contribution of this paper is a novel envelope process for modeling multiscaling traffic. The envelope process is an upper bound for the accumulated amount of traffic arrived up to a certain time from a multifractal Brownian motion process (mBm) [8]. It is shown that although mBm is a steady state Gaussian process, the envelope process is a tight bound to the amount of traffic arrived from real network streams. One of the advantages of this envelope process is the parsimonious representation of traffic, which allows a simple solution for queueing systems fed by multifractal streams. Moreover, an expression for the time at which a finite queueing system overflows is computed. Such expression can be used in admission control policies.

2 The Multifractional Brownian Motion Process

Multifractal processes exhibits highly irregular patterns as a function of time. Local Holder exponents describes the local regularity of the sample path of a process. It is a measure of scaling and can be regarded as a generalization of the Hurst parameter [1].

The local Holder regularity is related to scaling at small time scales since it expresses the regularity of the sample path of a process by comparing it to a power-law function [1]. The exponent of this power law, $H(t)$, is called Holder exponent and depends both on time and on the sample path. The Holder exponent is the largest value of $H(.)$, $0 \leq H(.) \leq 1$, such that

$$|X(t+\gamma) - X(t)| \leq k|\gamma|^{H(t)} \quad \text{for } \gamma \to 0 \tag{1}$$

where $X(.)$ is a process which exhibits scaling.

For monofractal processes the Holder function (Holder exponent) is a constant value and is called Hurst parameter whereas for multifractal processes the Holder function changes randomly with time. Let $H(.) : (0, \infty) \to (0, 1)$ be a Holder function. The multifractional Brownian motion is a continuous Gaussian process with non-stationary increments defined on $(0, \infty)$ as:

$$\begin{aligned} W_{H(t)} = \frac{1}{\Gamma(H(t)+1/2)} \Big\{ \int_{-\infty}^{0} [(t-s)^{H(t)-1/2} - \\ (-s)^{H(t)-1/2}] dB(s) + \int_{0}^{t} (t-s)^{H(t)-1/2} dB(s) \Big\} \end{aligned} \tag{2}$$

where $B(s)$ is the Brownian motion.

The multifractal Brownian motion process is a generalization of the fractal Brownian motion process and exhibits the nice property that locally it is asymptotically self-similar (lass), i.e.

$$\lim_{\rho \to 0^+} \left\{ \frac{W(t+\rho u) - W(t)}{\rho^{H(t)}} \right\}_{u \in R^+} = \{B_{H(t)}(u)\}_{u \in R^+} \tag{3}$$

where $W(.)$ is an mBm and $B_{H(t)}(u)$ is an fBm process with Hurst parameter H, given by $H(t)$.

Evaluating the Holder exponent value is crucial for the characterization of multifractal traffic. In [8] an estimator for the Holder exponent $H(.)$ was introduced. This estimator assumes that the Holder exponent is a continuous function and that its value is a constant in the neighborhoods of a point. For N data samples of an mBm $W(.)$, $H\left(\frac{i}{N-1}\right)$ can be estimated as [8]:

$$\hat{H}_{\frac{i}{N-1}} = \frac{-\log(\sqrt{\frac{\pi}{2}} S_{k,N}(i))}{\log(N-1)} \quad 1 \leq i \leq N-2, \tag{4}$$

where $S_{k,N}(i) = \frac{m}{N-1} \sum_{j \in [i-k/2, i+k/2]} \|W(j+1) - W(j)\|$ and $m = \frac{N}{k}$.

3 An Envelope Process for the Multifractal Brownian Motion Process

To solve a queueing system fed by an input process, it is necessary to know both the amount of work arrived to the system as well as the service rate up to a certain time. Envelope processes are upper bounds to the amount of arrivals. They can be either deterministic or probabilistic. In deterministic envelopes, the amount of work arrived never surpasses the envelope value whereas in probabilistic envelopes it may surpass with a certain pre-defined probability. Probabilistic envelope processes are tighter bounds than deterministic envelopes since deterministic envelope are always an upper bound and do not accept any violation of the envelope value. Consequently, dimensioning based on deterministic envelope processes may lead to waste of bandwidth, since the provision of bandwidth needs to take into account the maximum amount of work arrived at any time. When probabilistic envelopes are used, there is no need to consider spikes of work up to a certain amount defined by the probability of violation. However, loss of packets may occur.

An upper bound for the accumulated amount of work arrived can be computed as the mean amount of work plus an upper bound to the accumulated increments. An upper bound for mBm increments can be computed by using the upper bounds for the local fBm increments, since in the neighborhood of time t, an mBm can be approximated by an fBm with Hurst parameter $H(t)$. It is known that [7]:

$$Z_H(t) \leq \kappa H t^{H-1} \tag{5}$$

where $Z_H(t)$ is the fBm increments at time instant t.

As the size of local infinitesimal neighborhood of t goes to zero, the envelope process, $\hat{A}(t)$, of an mBm with mean $\bar{a}$, standard deviation σ and Holder function $H(.)$ can be expressed as :

$$\hat{A}(t) = \int_0^t \bar{a} + \kappa\sigma H(x) x^{H(x)-1} dx \tag{6}$$

which is called mBm envelope process.

This envelope reduces to the fBm envelope previously derived in [7] when $H(.)$ is a constant value, i.e.,

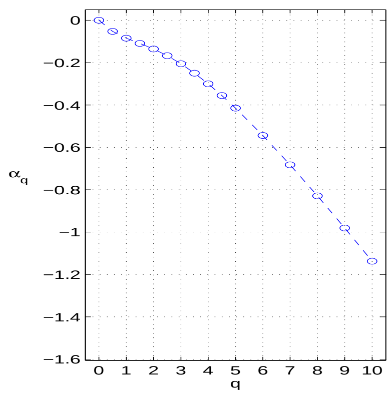
$$\hat{A}(t) = at + \kappa\sigma t^H \tag{7}$$

Extensive simulation experiments using both synthetic traffic and real network traffic were conducted in order to assess the accuracy of the proposed envelope. Traces containing real network traffic were obtained from the NLANR site (www.nlanr.net). These traces were collected at aggregation points in high performance connection networks, such as vBNS and Internet2 ABILENE. The sampling precision of the collection was of the order of microseconds. Table 1 shows the characteristics of the traces used in this investigation. Data shown in this paper correspond to the evaluation of traces MEM-1053844177 and MEM-1054459191 at time scale of 1msec.

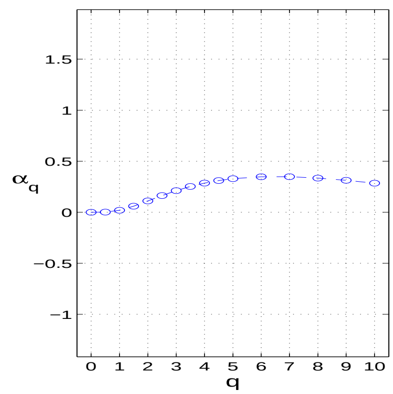
Multifractal analysis of the traces were pursued before its use. We used the code available on www.emulab.ee.mu.oz.au/darryl. Multifractality is detected by analyzing the scaling exponent $\alpha_q = \varsigma(q) + q/2$ in multiscale diagrams. A non-linear behavior indicates a dynamic $\varsigma(q)$ value. Figure 1 shows the multiscale analysis of traces MEM-1053844177 and MEM-1054459191. In both figures four distinct regions can be observed. For the trace MEM-1054459191 (Figure 1.a) the alignment of q values can be cast in the regions $[0-1]$, $[1-3]$, $[3-6]$ and $[6-10]$ whereas for the trace MEM-1053844177 (Figure 1.b) the distinct regions

Table 1: Traffic gather at vBNS and ABILENE networks

Trace	Date	packets	aggregation point
ANL-1050127417	04/11/03 23:12	121998	Agonne NL to STARTAP
ANL-1050225668	04/13/03 2:46	105641	Agonne NL to STARTAP
MEM-1053844177	05/24/03 23:54	220904	University of Memphis
MEM-1054459191	06/01/03 2:54	266708	University of Memphis
COS-1057970154	07/12/03 0:49	1247518	Colorado State University
BWY-1058086940	07/13/03 9:19	1168143	Columbia University



(a) Trace MEM-1054459191



(b) Trace MEM-1053844177

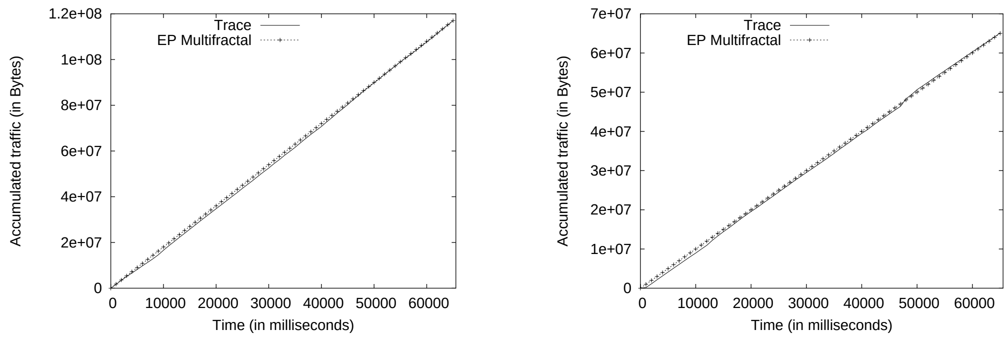
Figure 1: The multiscale diagrams for the traces MEM-1054459191 and MEM-1053844177

of q values are $[0-1]$, $[1-5]$, $[5-8]$ and $[8-10]$. The distinct patterns of α_q indicates the changing behavior of the Holder exponent and, consequently, non-trivial multifractality.

Figure 2 shows that the mBm envelope process provides a tight bound to the accumulated amount of traffic arrived in real networks. The mBm envelope process was also validated using the real network traces employed in [2] and in [10]. Similar precision to the results presented here was verified.

To answer the question of whether real network traces could be modeled by using a monofractal process, fBm envelope processes (Equation 7) were derived for the traces used. The Abry-Veitch Hurst estimator [12] was employed. Table 2 shows the parameters for the traces MEM-1053844177 and MEM-1054459191. Figure 3 show the accumulated real traffic and both the fBm envelope process (monofractal) and the mBm envelope process (multifractal). It can be observed in Figure 3 that the fBm envelope process deviates largely from the real trace. This happens because monofractal envelopes take into consideration only the global burstiness value, given by the Hurst parameter, which overestimates the dynamic (local) burstiness present in multifractal traffic. Figure 3 shows clearly that a monofractal model does not capture the dynamics of multifractal traffic.

The mBm generator introduced in [8] was used to generate synthetic data. Up to 10^6 data samples from mBm processes were generated. Different Holder exponent were

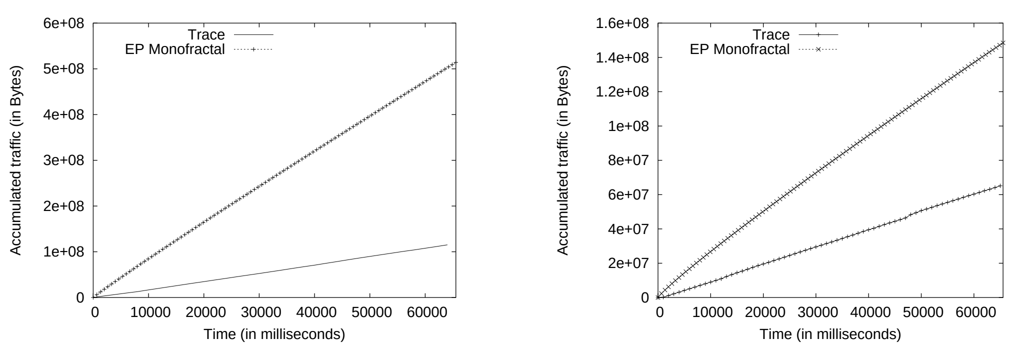


(a) Trace MEM-1054459191 (b) Trace MEM-1053844177

Figure 2: The mBm envelope process evaluation for real network traffic

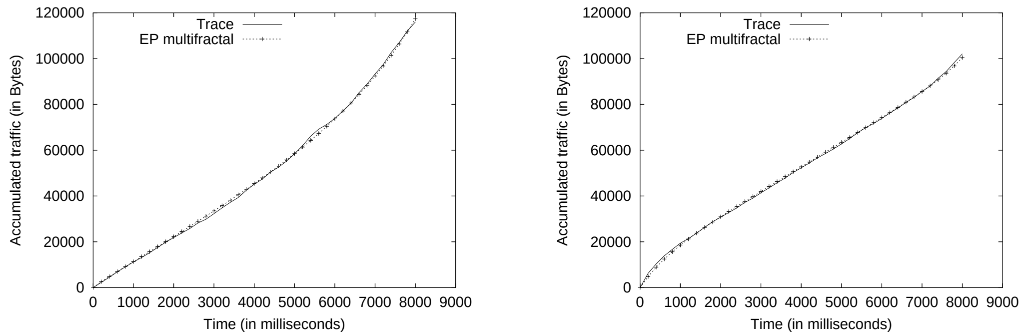
Table 2: The fBm Envelope Process parameters

trace	mean($\bar{a}$)	variance σ^2	H
MEM-1053844177	1000.8	3227339.6	0.85
MEM-1054459191	1788.9	8546650.0	0.94



(a) Trace MEM-1054459191 (b) Trace MEM-1053844177

Figure 3: The fBm envelope process evaluation for real network traffic



(a) H(.) a linear function

(b) H(.) a quadratic function

Figure 4: The mBm envelope process evaluation for synthetic traces

employed. In Figure 4, results are shown for the following Holder functions:

$$H(t) = 1.9 * (t - 0.5) * (t - 0.5) + 0.51 \quad t \in (0, 1);$$
$$H(t) = 0.5 + t/2.0 \quad t \in (0, 1). \tag{8}$$

It can be observed that the mBm envelope process is also a tight bound to the modeled processes regardless of the Holder exponent values. Violations of the established bound are smaller than the pre-defined violation probability value.

4 Time Scale of Interest

In this section the time at which a queue reaches its maximum occupancy in a probabilistic sense is derived. The queue size at this time provides a simple delay bound. Consider a continuous-time queueing system, with deterministic service given by C. The cumulative arrival process is represented by $A_{H(t)}(t)(A_{H(t)}(0) = 0)$. Let $\hat{A}_{H(t)}(t)$, a function continuous and differentiable, be the probabilistic envelope process of $A_{H(t)}(t)$, such that $P(A_{H(t)}(t) > \hat{A}_{H(t)}(t)) \leq \epsilon$.

During a busy period, which starts at time 0, the number of cells in the system at time t is given by $q(t)$. Thus, $q(t) = A_{H(t)}(t) - ct \geq 0$.

By defining $\hat{q}(t)$ as

$$\hat{q}(t) = \hat{A}_{H(t)}(t) - Ct \geq 0, \tag{9}$$

we can see that $P(q(t) > \hat{q}(t)) = P(A_{H(t)}(t) > A_{H(t)}(t)) \leq \epsilon$.

The maximum delay in a FIFO queue is given by the maximum number of cells in the queue during the busy period. An upper bound for the maximum delay in a FIFO queue is giving by

$$\hat{q}_{max} = max(\hat{q}(t)) \qquad t \geq 0 \tag{10}$$

Therefore,

$$P(q(t) > \hat{q}_{max}) \leq P(q(t) > \hat{q}(t)) \leq \epsilon \tag{11}$$

$$P(q(t) > \hat{q}_{max}) \approx \epsilon. \tag{12}$$

The queue length at time t, $q(t)$, will only exceed the maximum queue length $\hat{q}_{max}$ with probability ϵ. In other words, only when the arrival process exceeds the envelope process, will the maximum number of cells in the system exceed the estimated value. Intuitively, by bounding the behavior of the arrival process, it is possible to transform the problem of obtaining a probabilistic bound for the stochastic system defined by $q(t) = A_{H(t)}(t) - Ct$, into an easier problem of finding the maximum of a deterministic system, described by $\hat{q}(t) = \hat{A}_{H(t)}(t) - Ct$.

The mBm process is inserted into Equation 9 giving:

$$\hat{q}(t) = \hat{A}_{H(t)}(t) - Ct = \int_0^t \bar{a} + \kappa\sigma H(x) x^{H(x)-1} dx - Ct \tag{13}$$

In order to compute $\hat{q}_{max}$, it is necessary to find $t^\star$ such that

$$\frac{d\hat{q}(t)}{dt} = 0 \tag{14}$$

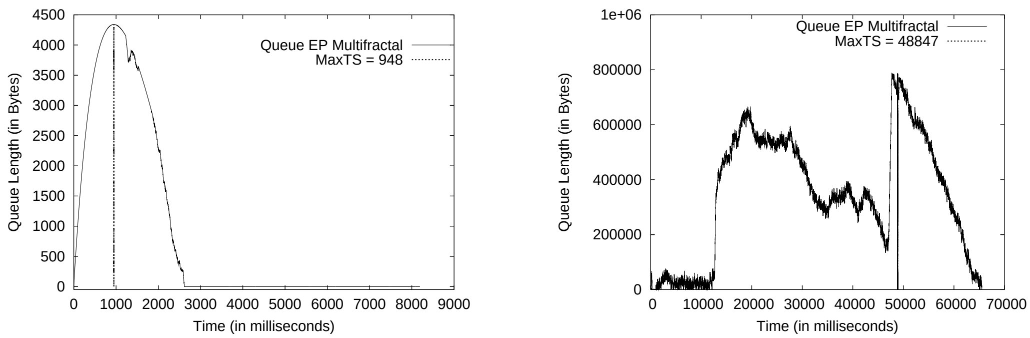
or equivalently,

$$\frac{d\hat{A}(t)}{dt} = C \tag{15}$$

The time-scale of interest, $t^\star$, is the time at which the queue size reaches its peak, called the Maximum Time-Scale (MaxTS) and $t^\star$ defines the point in time when the unfinished work in the queue achieves its maximum in a probabilistic sense. Hence, $t^\star$ can be computed from Equation 15 as:

$$t^\star = \left[\frac{\kappa\sigma H(t^\star)}{(C - \bar{a})}\right]^{\frac{1}{1-H(t^\star)}} \tag{16}$$

To evaluate the precision of the expression for the time scale of interest, simulation experiments were conducted. A queue with constant service rate was fed by an mBm and the queue length was recorded. The estimated time scale of interest was computed and compared to the time at which the queue length reaches its maximum in the simulation experiments. Figure 5 shows results for an mBm process with a polynomial quadratic Holder function (Equation 8) as well as for real network trace. The computed value, MaxTS=948, is the same found in the simulation experiment for the synthetic multifractal process (Figure 5.a). Figure 5.b shows the time scale of interest for the experiment using the trace MEM-1053844177. It can be seen that the estimated time scale of interest (MaxTS=48847) closely



(a) H() a quadratic function

(b) Trace MEM-1053844177

Figure 5: MaxTS for both synthetic and real network traffic

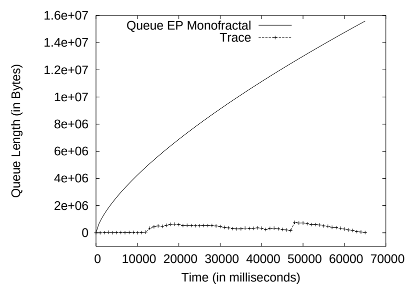


Figure 6: MaxTS estimated using monofractal EP for real network traffic

approximates the one found in the simulation experiments. Note that, such deviation is within the know error margin established by the violation probability value.

In order to emphasize that a monofractal approach is inappropriate for approximating the behavior of multifractal traffic, the time scale of interest is computed considering a monofractal envelope process for the trace MEM-1053844177 (Table 2). The expression for the time scale is given by [7]:

$$t^\star = \left[\frac{\kappa \sigma H}{(C - \bar{a})} \right]^{\frac{1}{1-H}} \tag{17}$$

The computed $t^\star$ value is $4.27 * 10^{18}$ for $C = 1.01 * \bar{a}$. It can be observed (Figure 6) that the time scale computed using a monofractal model is far away from the one obtained via simulation. If time scales derived by monofractal models were used for dimensioning networks with multi-scaling traffic, resources would be greatly overestimated leading to bandwidth waste.

5 Related work

A. Erramilli, O. Narayan, A. Neidhardt and I Saniee [2] proposed that traffic should be modeled by random cascades at time scales smaller than a cutoff value and be represented by an fBm at larger scales. They show that for IP traffic, the cutoff scale is of the order of one Round Trip Time (RTT), while for VBR video it is typically of the order of a frame duration. Erramili et. al. showed that much more accurate results can be obtained by using their model rather than using purely monofractal models.

Other models based on multiplicative cascade have been proposed. These models map a given sample into a binary multiscale tree [11]. Each node in the tree corresponds to the aggregation of the traffic mapped into its descendents. Thus, nodes at higher levels of the tree correspond to coarser time scale whereas nodes at lower levels correspond to finer time scales. The multipliers (weights) assigned to each descendent of a node can be set to represent a specific marginal distribution and scaling. In the wavelet-domain independent model (WIG) [6], multipliers are independent additive innovations and correspond to the Haar wavelet coefficient of the process represented by the binary multiscaling tree. As the depth of the tree goes to infinite, the marginal traffic distribution tends to a Gaussian. In the Multifractal Wavelet Model (MWM) [10], multipliers are multiplicative innovations, generating a log-normal marginal distribution, approximately. Both models require the setting of $2 + log_2 N$ parameters where N is the sample size. It has been shown that MWM captures more precisely the dynamics of real traces than WIG. The major drawback of these models, however, are the number of parameters to be fitted. Moreover, they require the construction of multiscaling binary tree which is not suitable for on-line characterization. These aspects prevent the use of these models for real-time bandwidth management since the parameters of processes resulting from the aggregation of distinct traffic streams need to be computed on-line.

Recent investigation [13] on small time scales of Internet traffic points out that monofractal behavior is observed at these scales. It is claimed that correlations at small time scales are caused mainly by flows with bursts of densely clustered packets and not by the acknowledgement mechanism of TCP. However, in our investigations using publicly available traces we found clear multifractal behavior at these scales(Figure 1).

6 Conclusions

Scaling analysis of IP and video traffic have pointed out the multifractal nature of the type of traffic [2] [10]. Models based on multiscaling have been proposed in the literature [11] [10]. These models, however, need the knowledge of the whole stream beforehand. Moreover, the number of parameters to be fitted depends on the sample size.

In this paper, a novel probabilistic envelope process for multifractal traffic modeling was introduced. The envelope process is an upper bound to the amount of work arrived from a multifractal Brownian motion process. Extensive simulation experiments using both synthetic and real network traffic show that the proposed model is a tight bound to the modeled traffic. Expressions for the time scale of interest was derived. Moreover, the inappropriateness of monofractal models for multifractal traffic modeling was emphasized.

Currently, a comparison between the precision of the time scale of interest derived using the mBm envelope process and the one derived using the multifractal wavelet model [10] is under investigation. Moreover, comparisons between the loss probability predicted by measurement-based models [3] [9] and by the mBm envelope process are also under investigation.

Acknowledgments

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